A General Formula for the Optimal Level of Social Insurance

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MOTIVATION

- Basic PF question: What is the optimal amount of social insurance?

- Canonical analysis of this question is due to Baily (1978)

- Baily's contribution: Simple "reduced-form" expression for optimal unemployment insurance benefit rate in terms of three parameters:

  1. Benefit elasticity of unemployment durations

  2. Consumption-smoothing benefit of UI

  3. Coefficient of relative risk aversion

- Subsequent empirical literature has been guided by this result

  - Moffitt (1985), Meyer (1990) estimate duration elasticity

  - Gruber (1997), Browning and Crossley (2001) estimate consumption-smoothing benefits

MOTIVATION

- However, many theoretical studies have emphasized limitations of Baily's simple static model

  - Liquidity constraints (Flemming 1978; Crossley and Low 2005)

  - Dynamic search (Lentz, 2004)

  - Human capital accumulation effects (Brown and Kaufold 1988)

- Baily's analysis also omits other potentially relevant factors:

  - Endogenous private insurance (Golosov and Tsyvinski 2005)

  - Leisure benefits of unemployment (Gruber 1997)

  - Inadequate model of savings (Feldstein 2005)

GOAL

- These studies raise concern that Baily's simple intuition is not widely applicable

  - But have not examined whether a Baily-type formula holds in the more general environment they advocate

- Today's paper provides reduced-form formulas for optimal benefit level and welfare gain from SI in a general dynamic stochastic model

- Preview of main result: Baily-type result holds in a very general class of models subject to weak regularity conditions
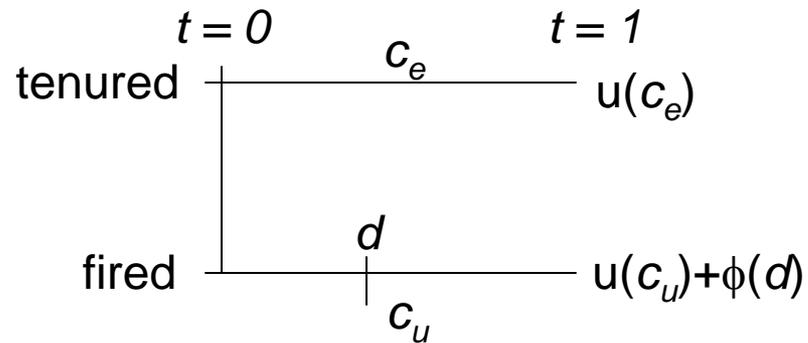
OUTLINE

I)   Special case

II)   General case

III)   Drawback of reduced-form method

IV)   Conclusion

BASIC FRAMEWORK

- Consider an unemployment insurance system that pays constant benefit *b* to unemployed agents

  - Optimal *path* of benefits not considered here

- Benefits financed by lump-sum tax τ on employed agents (so government budget is balanced in expectation in each period)

- Some important assumptions maintained throughout:

  1. Wages fixed (no GE effects)

  2. No distortions to firm behavior (temporary layoffs); assume perfect experience rating

  3. No externalities (e.g. spillovers to search)

SPECIAL CASE: TENURE REVIEW MODEL

- Agent lives for one unit of (continuous) time

- Arrives at time 0 (tenure review) with assets A_0

- Two states: Tenured at $t = 0$ (probability p) or fired (1-p)

- Assume p exogenous (independent of b) in special case

- In employed state, no risk of job loss until death (tenure), and agent makes no labor supply choices

- If fired, agent must search for a job

- Agent can control unemp duration d deterministically by varying search effort

- Search costs, matching benefits of search, etc. captured by a concave, increasing function $\phi(d)$ that enters utility additively

$t = 0$ $t = 1$
                          c_e
tenured ├──────────────────────┤ $u(c_e)$

                          d
fired   ├───────────────┼──────┤ $u(c_u)+\phi(d)$
                          c_u

- Only constraints are budget constraint in each state

- In special case, assume UI tax collected only in tenured state to simplify algebra

- Normalize interest rate and discount rate to 0

- No uncertainty and no discounting implies optimal consumption path is flat in each state

    c_e = cons. in employed (tenured) state
    c_u = cons. in unemp (fired) state

- Let u(c) = utility over consumption (strictly concave)

Agent's Problem

$$\max(1-p)u(c_e) + p\{u(c_u) + \psi(d)\}$$
$$\text{s.t.}\ A_0 + (w - \tau) - c_e \geq 0$$
$$A_0 + bd + w(1-d) - c_u \geq 0$$

Let $V(b)$ denote the solution to this problem (indirect utility).

Planner's Problem

$$\max_b V(b)$$
$$\text{s.t.}\ (1-p)\tau = pbd$$

Goal: Simple, empirically implementable characterization of b^*

At an interior optimum, optimal benefit rate must satisfy

$$\partial V/\partial b(b^*) = 0$$

We can write

$$V(b) = \max_{c_e,c_u,d,\lambda_e,\lambda_u} (1-p)u(c_e) + p\{u(c_u) + \psi(d)\}$$
$$+ \lambda_e[A_0 + (w-\tau) - c_e] + \lambda_u[A_0 + bd + w(1-d) - c_u]$$

Here λ_e and λ_u represent marginal value of relaxing BC in each state.

Envelope Theorem: $\partial V/\partial x = 0$ for $x \in \{c_e, c_u, d, \lambda_e, \lambda_u\}$. Therefore

$$\partial V/\partial b(b^*) = -\lambda_e \frac{\partial \tau}{\partial b} + \lambda_u d = 0 \tag{1}$$
$$\Rightarrow \lambda_e \frac{\partial \tau}{\partial b} = \lambda_u d \tag{2}$$

Agent optimization implies that the multipliers are equal to the marginal utility of consumption in each state:

$$\lambda_e = (1-p)u'(c_e) \tag{3}$$
$$\lambda_u = pu'(c_u) \tag{4}$$

Government's UI budget constraint implies

$$\frac{\partial \tau}{\partial b} = \frac{p}{1-p}[d + b\frac{\partial d}{\partial b}]$$

At b^*,

$$\lambda_e \frac{\partial \tau}{\partial b} = \lambda_u d$$
$$\Rightarrow u'(c_e)[1 + \frac{b}{d}\frac{\partial d}{\partial b}] = u'(c_u)$$

$$u'(c_e)[1 + \frac{b}{d}\frac{\partial d}{\partial b}] = u'(c_u) \tag{5}$$

- This condition captures a central intuition that holds in general case

  – Optimal level of benefits offsets the MB of raising c_u by \$1 (RHS) against the MC of raising τ in the employed state to cover the added benefits (LHS).

  – To finance \$1 of extra c_u, UI tax must be increased by \$1 + added amount to cover agent's behavioral response of extending unemployment duration, which reduces c_u.

Rearranging (5), we obtain

$$\frac{u'(c_u) - u'(c_e)}{u'(c_e)} = \frac{b}{d}\frac{\partial d}{\partial b} = \varepsilon_{d,b} \tag{6}$$

This equation provides an exact definition for b^*, for a given u.

Simplify this expression further using Taylor approximation:

$$u'(c_u) - u'(c_e) \approx u''(c_e)(c_u - c_e) + \frac{1}{2}u'''(c_e)(c_e - c_u)^2.$$

Define $\gamma = -\frac{u''(c_e)}{u'(c_e)}c_e$ (risk aversion) and $\rho = -\frac{u'''(c_e)}{u''(c_e)}c_e$ (prudence)

Letting $\Delta c/c == \frac{c_e - c_u}{c_e}$, it follows that

$$\frac{u'(c_u) - u'(c_e)}{u'(c_e)} \approx \gamma(\frac{\Delta c}{c} + \frac{1}{2}\rho\frac{\Delta c}{c})$$

Proposition 1: If fourth-order terms of $u(c)$ are small $(u''''(c) \approx 0)$,

$$\gamma \frac{\Delta c}{c}(b^*)[1 + \frac{1}{2}\rho\frac{\Delta c}{c}(b^*)] \approx \varepsilon_{d,b} \qquad (7)$$

If third-order terms of $u(c)$ are small $(u'''(c) \approx 0)$,

$$\gamma \frac{\Delta c}{c}(b^*) \approx \varepsilon_{d,b} \qquad (8)$$

- Intuition: cons-smoothing benefits $=$ efficiency cost of UI

- Three parameter formula is identical to Baily's (1978) expression

  – Problem is that second-order approx of u sometimes understates optimal b^* by $>30\%$ for parameters used by Gruber (1997).

  – Error in third-order approx for CRRA utility is less than 4%.

- **Intuition underlying derivation of Proposition 1**

  - Higher benefits simply relax budget constraint

  - Agent has already equated marginal utilities at optimum, so we can assume when calculating welfare change that extra b is spent solely on c_u

  - Analogously, can assume higher tax financed solely by reducing c_e

  - All other behavioral responses can be ignored, and welfare change from UI can be expressed simply through $u'(c_e)$ and $u'(c_u)$

  - Only additional term is $\varepsilon_{d,b}$ which determines government's budget requirement ($\partial\tau/\partial b$ term)

General Case

- Now show that this formula holds in a general environment

- Continuous-time dynamic model where agents face persistent unemployment risk

- Normalize the length of life to be one unit: $t \in [0, 1]$

- Let $c_e(t)$ denote consumption if employed at t and $c_u(t)$ if unemployed

- Agents choose a vector of other behaviors in each state

$$x_e(t) = (x_e^1(t), ..., x_e^{M_e}(t)) \text{ and } x_u(t) = (x_u^1(t), ..., x_u^{M_u}(t))$$

  – search effort or reservation wage while unemployed

  – level of work effort (or shirking) while employed

  – private insurance purchases

  – amount of borrowing from friends

  – portfolio choice

  – human capital investments

- Let $u(c_t, x_t)$ denote the felicity utility at t

- Let $c = \{c_e(t), c_u(t)\}_{t=0}^{1}$ and $x = \{x_e(t), x_u(t)\}_{t=0}^{1}$ denote full program of choices over time.

- Let $\theta_t(c, x, t)$ denote employment status at time t.

  - $\theta(c, x, t) = 1$ -> employed at t; $\theta(c, x, t) = 0$ -> unemployed

- Process that determines $\theta(c, x, t)$ left unspecified,

  - Can be an arbitrary function of the agent's behavior at time t as well as other times.

  - Trajectory of θ stochastic, with a general, time-varying disturbance term

- Let d denote the fraction of lifetime spent in unemployed state:

$$d = \int_0^1 [1 - \theta(t)]dt$$

- Let $\widehat{c_e}$ and $\widehat{c_u}$ denote mean consumption in each state:

$$\widehat{c_e} = \frac{\int \theta(t)c_e(t)dt}{\int \theta(t)dt}$$

$$\widehat{c_u} = \frac{\int (1 - \theta(t))c_u(t)dt}{\int (1 - \theta(t))dt}$$

Constraints

- Standard dynamic budget constraint while employed and unemployed

$$\dot{A_e}(t) = w - \tau - c_e(t) \; \forall t \qquad (9)$$
$$\dot{A_u}(t) = b - c_u(t) \; \forall t$$

- Terminal condition on assets:

$$A(1) = A_0 + \int_0^1 [\theta(t)\dot{A_e}(t) + (1 - \theta(t))\dot{A_u}(t)]dt \geq A_{term}$$

- In addition, the agent faces a set of N additional constraints at each time t

$$g_{it}(c_\theta(t), x_\theta(t); b, \tau) \geq k_{it}, i = 1, ..., N; \theta = 0 \text{ or } 1$$

Agent's Problem

$$\max \int_0^1 \theta(t)u_t(c_e(t), x_e(t)) + (1 - \theta(t))u_t(c_u(t), x_u(t))dt$$

s.t. constraints

Let $V(b)$ denote the solution to this problem (indirect utility).

Planner's Problem

$$\max_b V(b)$$

$$\text{s.t.} \qquad \tau \int \theta(t)dt = b \int [1 - \theta(t)]dt$$

$$\implies \tau(1 - d) = db$$

- Main result: Simple formula for b^* generalizes under weak conditions

- Regularity conditions to ensure a unique interior optimum (smoothness and quasiconcavity of utility, convexity of choice set)

- Key assumption: Consumption-UI Constraint Condition

  - Must be able to quantify the costs and benefits of unemployment insurance solely through $u'(\widehat{c_e})$ and $u'(\widehat{c_u})$

  - Feasible if higher benefits relax all constraints on consumption while unemployed and higher taxes tighten all constraints on consumption while employed.

Assumption 5. The feasible set of choices can be defined using a set of constraints $\{g_{it}\}$ such that $\forall t \forall i$

$$\text{(a)} \quad \frac{\partial g_{it}}{\partial b} = -\frac{\partial g_{it}}{\partial c_u(t)}$$

$$\text{(b)} \quad \frac{\partial g_{it}}{\partial \tau} = \frac{\partial g_{it}}{\partial c_e(t)}$$

$$\text{(c)} \quad \frac{\partial g_{it}}{\partial c_\theta(s)} = 0 \text{ if } t \neq s$$

(a) benefits and consumption while unemployed enter each constraint in the same way

(b) the UI tax and consumption while employed enter each constraint in the same way

(c) consumption at two different times s and t do not enter the same constraint together

Examples

1. Budget constraints.

$$\dot{A}_e(t) = w - \tau - c_e(t) \ \forall t$$
$$\dot{A}_u(t) = b - c_u(t) \ \forall t$$

- $\frac{\partial \dot{A}_u(t)}{\partial b} = -\frac{\partial \dot{A}_u(t)}{\partial c_u(t)} = 1$ and $\frac{\partial \dot{A}_e(t)}{\partial \tau} = \frac{\partial \dot{A}_e(t)}{\partial c_e(t)} = -1$. Since only $c_\theta(t)$ appears in each constraint at time t, assumption 5 holds.

2. Borrowing constraint while unemployed at time t.

$$g_{1t} = A(t) + b - c_u(t) \geq 0$$

- $\frac{\partial g_{1t}}{\partial b} = -\frac{\partial g_{1t}}{\partial c_u(t)} = 1$ and $\frac{\partial g_{1t}}{\partial \tau} = \frac{\partial g_{1t}}{\partial c_e(t)} = 0 \ \forall t$, so assumption 5 holds.

3. Private insurance market.

- Private insurance contract that charges a premium $\rho_e(t)$ in the employed state

- Net payout of $\rho_u(t)$ in the unemployed state at time t.

- Changes the dynamic budget constraints to:

$$
\begin{aligned}
\dot{A}_e(t) &= w - \rho_e(t) - \tau - c_e(t) \ \forall t \\
\dot{A}_u(t) &= b + \rho_u(t) - c_u(t) \ \forall t
\end{aligned}
$$

- $\frac{\partial \dot{A}_u}{\partial b} = -\frac{\partial \dot{A}_u}{\partial c_u(t)} = 1$ and $\frac{\partial \dot{A}_e}{\partial \tau} = \frac{\partial \dot{A}_e}{\partial c_e(t)} = -1$, etc. so assumption 5 holds.

4. Subsistence constraint. Suppose the agent must maintain consumption above a level $\underline{c}$ at all times:

$$g_{3t} = c_\theta(t) - \underline{c} \geq 0 \ \forall \theta, t$$

- If this constraint binds, $\frac{\partial g_{3t}}{\partial b} = 0 \neq \frac{\partial g_{3t}}{\partial c_\theta(t')} = 1$, so assumption 5 is *not* satisfied here.

- Intuitively, subsistence constraint cannot be loosened by providing more UI benefits, so benefit of UI can no longer be read from consumption change.

- Subsistence constraint represents a pathological case.

  - Most agents are able to cut consumption when benefits are lowered in practice (Gruber 1997).

  - Marginal utility of consumption should rise toward infinity as consumption falls to $\underline{c}$, preventing constraint from binding.

- More generally, as long as different sources of income are fungible, higher b should permit higher c_u

  - Only reason this might not be feasible is because of other technological constraints on consumption.

  - Most plausible constraints do not involve such restrictions

- Given this assumption, simple formula for b^* obtained as above:

  1. Write $\partial V/\partial b$ in terms of multipliers on constraints using Envelope condition

  2. Write multipliers in terms of $u'(c_u)$ and $u'(c_e)$ by exploiting constraint condition

  3. Take Taylor approximations as above and simplify

Proposition 2.

In general environment, b* is approximately defined by

$$\frac{\Delta c}{c}(b^*)\gamma[1 + \frac{1}{2}\rho\frac{\Delta c}{c}(b^*)] \approx \frac{\varepsilon_{d,b}}{1-d} \qquad (10)$$

Change in welfare from an increase in b relative to the change in welfare from a permanent increase in consumption while employed is approximately

$$\frac{\partial V/\partial b}{(1-d)Eu'(c_{e,t})} \approx \frac{d}{1-d}[\frac{\Delta c}{c}(b)\gamma[1 + \frac{1}{2}\rho\frac{\Delta c}{c}(b)] - \frac{\varepsilon_{d,b}}{1-d}] \qquad (11)$$

Formula is identical to that in simple "tenure review" model except in two respects.

1. Inputs reflect *average* behavioral responses over time.

   - Consumption drop is the percentage difference between *average* consumption while employed and unemployed ($\Delta c / c = \frac{\widehat{c}_e - \widehat{c}_u}{\widehat{c}_e}$)

   - $\varepsilon_{d,b}$ term is the effect of a 1% increase in b on the fraction of his life the agent spends unemployed

2. Added $\frac{1}{1-d}$ term that scales up $\varepsilon_{d,b}$.

   - Because raising c_u by \$1 now also causes a reduction in tax collection since the agent spends less time employed

   - Small effect because d close to 0 in practice

Key implication of this result: Most of the generalizations proposed since Baily (1978) do not require reformulation of benefit rule!

1. Borrowing constraints. Tighter borrowing constraints will make us observe a larger consumption-smoothing effect in the data and raise b^*

2. Endogenous insurance markets. Again captured through the $\frac{\Delta c}{c}$ parameter, which will be smaller if agents have made private market arrangements

3. Multiple consumption goods. Sufficient to obtain consumption smoothing estimates for a single good (e.g., food)

- All other consumption goods can be placed in the set of x other choice variables

- PSID data limitations not a concern

- Durability of consumption not a problem

  - Browning and Crossley (2003) on socks and stocks

4. Search or leisure benefits of unemployment.

   - Workers internalize this: If they recognize large gains to search or leisure, they will choose a long unemployment spell and large $\frac{\Delta c}{c}$

   - Drives up b^* in reduced-form formula

   - Only consumption-smoothing benefits need to be explicitly considered

5. Dynamic search and savings behavior (e.g. Lentz 2004)

   - Nested within general case considered here

   - Will not change conclusions about b^* if it is calculated using (10) even though varying structural parameters changes results

- Main concept: These generalizations simply change values of key inputs $(\gamma, \frac{\Delta c}{c}, \text{ and } \varepsilon_{d,b})$.

  – If these are estimated directly from data, underlying process and structure that generated them is irrelevant in calculating b^*

- Reduced-form formula provides a simple yet robust means of making normative statements about social insurance

Drawback of Reduced-Form Method

- Restrictions imposed by other data and parameters may be ignored because they enter formula implicitly

- Example: Formula derived here suggests that total (uncompensated) elasticity $\varepsilon_{d,b}$ matters for b^*

  – But UI is essentially a state-contingent tax system

  – Tax theory intuition is that efficiency cost of UI is determined only by pure substitution effect (not income effect)

- Why does this not come through here?

- Can show that γ is a function of income and substitution elasticity

  – Large income elasticity directly implies γ large, so b^* rises

- Danger is that one may calibrate formula with low values of γ without recognizing restriction on income effect

  – Accumulating evidence that income effects in unemployment are actually large (Cullen and Gruber 1998, Chetty 2005)

- This is just one example of many implicit restrictions not apparent in reduced-form approach

- Therefore very useful to estimate other behavioral responses to UI to check if three main parameters are consistent

Conclusion

- A simple formula for welfare gains and optimal level of social insurance in a wide class of stochastic dynamic models

  – Formula can easily be modified to analyze other policies (e.g. disability insurance or welfare programs)

- Reduced-form empirical estimates of behavioral responses very useful

  – But structural tests needed to determine consistency of parameters

Directions for Further Research

1. Endogenous takeup decision

2. Myopic agents and internalities/externalities

3. General equilibrium effects